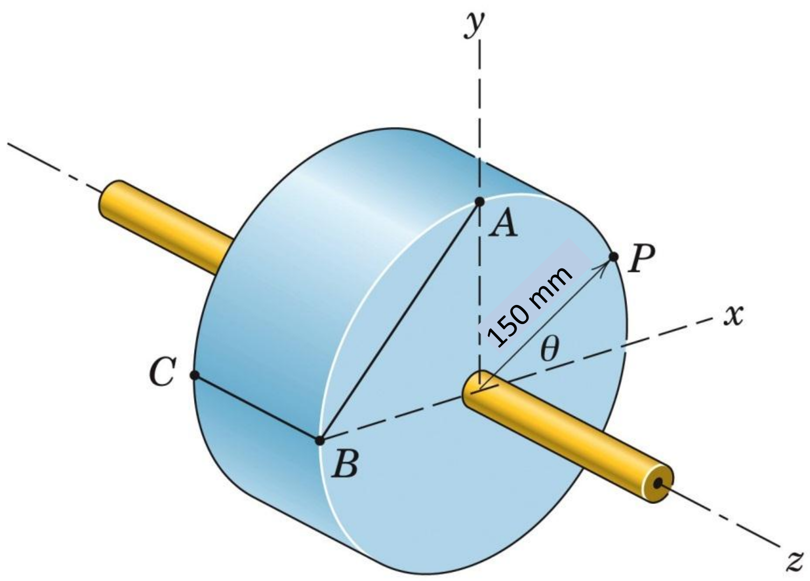
# ÇANKAYA UNIVERSITY – MECHANICAL ENGINEERING DEPARTMENT
# ME 204 – DYNAMICS – SPRING 2013
# HOMEWORK 4
# PLANE KINEMATICS OF RIGID BODIES

*Due Date: 1st Lecture Hour of Week 11*

**PROBLEM 5/25**

The solid cylinder rotates about its z-axis. At the instant represented, point P on the rim has a velocity whose x-component is -1.28 m/s, and $\theta = 20^0$. Determine the angular velocity ω of line AB on the face of the cylinder. Does the element line BC have an angular velocity?

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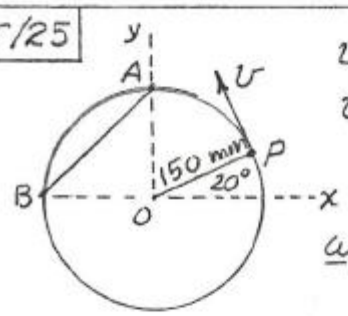



$v_x = -1.28$ m/s

$v = 1.28/\sin 20° = 3.74$ m/s

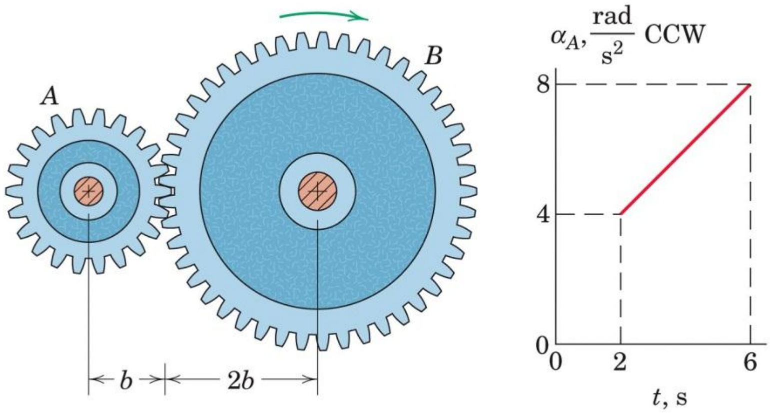
$\omega = \frac{v}{r} = \frac{3.74}{0.150} = 24.9$ rad/s CCW

$\underline{\omega} = \underline{\omega}_{AB} = \underline{\omega}_{OP} = \underline{24.9\, \underline{k}}$ rad/s

Element BC remains parallel to z-axis so has no angular velocity.

PROBLEM 5/28

The design characteristics of a gear-reduction unit are under review. Gear $B$ is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear $A$ at time $t = 2$ s to give gear $A$ a counterclockwise acceleration $\alpha$ which varies with time for a duration of 4 seconds as shown. Determine the speed $N_B$ of gear $B$ when $t = 6$ s.



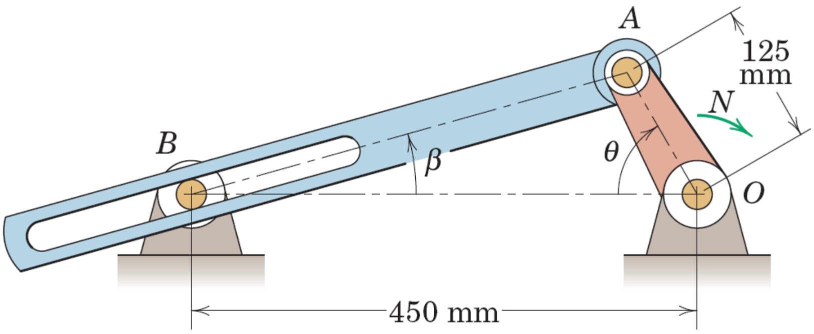

For gear $A$, $\Delta\omega = \int_2^6 \alpha_A \, dt$, $N_A = 2N_B$

$$(N_A - 600)\frac{2\pi}{60} = \frac{4+8}{2}(6-2), \quad N_A = 600 + 229 = 829 \text{ rev/min}$$

so at $t = 6$ s, $N_B = \frac{829}{2} = \underline{415 \text{ rev/min}}$

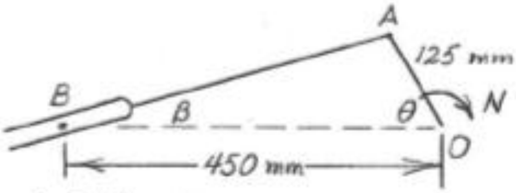
PROBLEM 5/53

Angular oscillation of the slotted link is achieved by the crank $OA$, which rotates clockwise at the steady speed $N = 120$ rev/min. Determine an expression for the angular velocity $\dot{\beta}$ of the slotted link in terms of $\theta$.




5/53

$$\tan\beta = \frac{125\sin\theta}{450-125\cos\theta}$$




$$\dot{\theta} = \frac{2\pi N}{60} = \frac{120}{30}\pi = 12.57 \text{ rad/s}$$

$$\sec^2\beta\,\dot{\beta} = \frac{(450-125\cos\theta)125\dot{\theta}\cos\theta - 125\sin\theta(125\dot{\theta}\sin\theta)}{(450-125\cos\theta)^2}$$

$$= \frac{56\,250\cos\theta - 15\,625}{(450-125\cos\theta)^2}\dot{\theta}$$

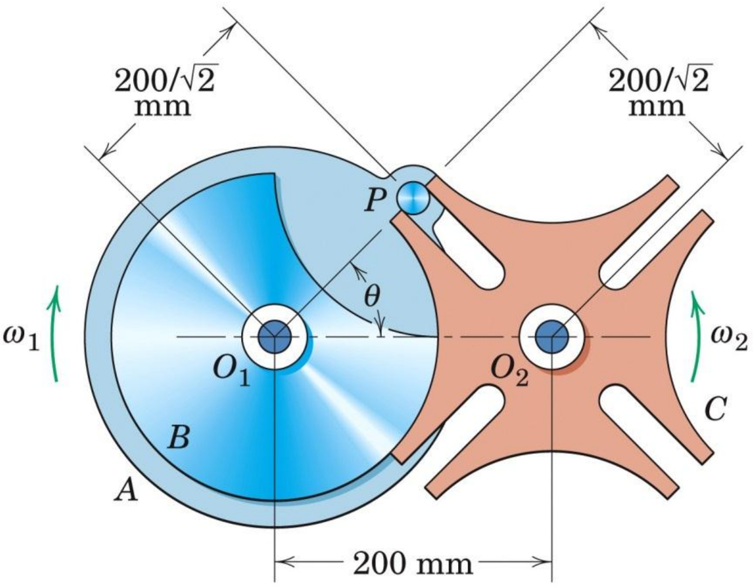
$$\dot{\beta} = \frac{56\,250\cos\theta - 15\,625}{(450-125\cos\theta)^2}\dot{\theta}\cos^2\beta$$

But $\cos^2\beta = \dfrac{(450-125\cos\theta)^2}{450^2+125^2-2(450)(125)\cos\theta}$

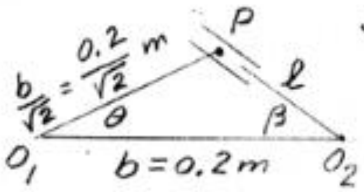
so $\dot{\beta} = \dfrac{56\,250\cos\theta - 15\,625}{218\,125 - 112\,500\cos\theta}12.57$ or $\underline{\dot{\beta} = \dfrac{12.57}{2}\dfrac{\cos\theta - 0.278}{1.939-\cos\theta}\text{ rad/s}}$

PROBLEM 5/56

The Geneva wheel is a mechanism for producing intermittent rotation. Pin $P$ in the integral unit of wheel $A$ and locking plate $B$ engages the radial slots in wheel $C$, thus turning wheel $C$ one-fourth of a revolution for each revolution of the pin. At the engagement position shown, $\theta = 45°$. For a constant clockwise angular velocity $\omega_1 = 2$ rad/s of wheel $A$, determine the corresponding counterclockwise angular velocity $\omega_2$ of wheel $C$ for $\theta = 20°$. (Note that the motion during engagement is governed by the geometry of triangle $O_1O_2P$ with changing $\theta$.)




▸5/56 $\frac{b/\sqrt{2}}{\sin\beta} = \frac{b}{\sin(\pi-\theta-\beta)} = \frac{b}{\sin(\theta+\beta)}$




so $\sqrt{2}\sin\beta = \sin(\theta+\beta)$ - - - - - (a)

$\sqrt{2}\,\dot\beta\cos\beta = (\dot\theta+\dot\beta)\cos(\theta+\beta)$

$\omega_2 = -\dot\beta = \dot\theta\,\frac{\cos(\theta+\beta)}{\cos(\theta+\beta)-\sqrt{2}\cos\beta}$ - - - (b)

From (a) $\sin\beta(\sqrt{2}-\cos\theta) = \sin\theta\cos\beta$, $\tan\beta = \frac{\sin\theta}{\sqrt{2}-\cos\theta}$

For $\theta = 20°$, $\beta = \tan^{-1}\frac{0.3420}{\sqrt{2}-0.9397} = \tan^{-1}0.7208 = 35.8°$

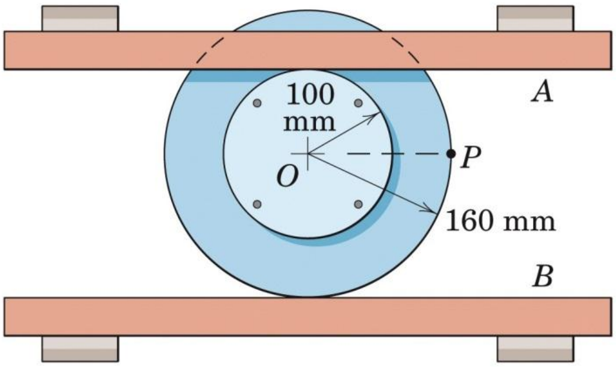
& for $\dot\theta = -2$ rad/s, Eq.(b) gives

$$\omega_2 = -2\,\frac{\cos(20°+35.8°)}{\cos(20°+35.8°)-\sqrt{2}\cos 35.8°} = -2\,\frac{0.5623}{-0.5849}$$

$\omega_2 = 1.923$ rad/s

PROBLEM 5/75

Each of the sliding bars $A$ and $B$ engages its respective rim of the two riveted wheels without slipping. Determine the magnitude of the velocity of point $P$ for the position shown.

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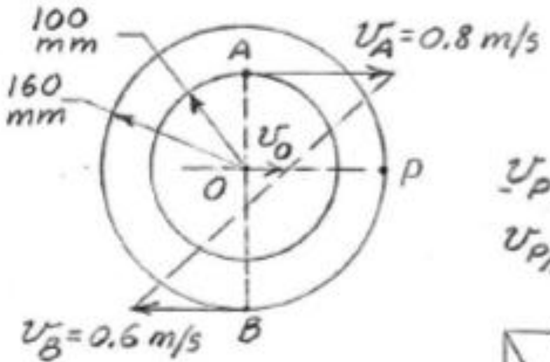
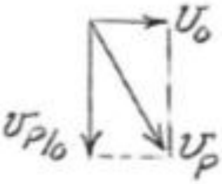



$$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8 + 0.6}{0.26} = 5.38 \ \frac{\text{rad}}{\text{s}} \ \text{CW}$$

$$v_O = v_A - \overline{AO}\,\omega$$
$$= 0.8 - 0.1(5.38) = 0.262 \ \frac{\text{m}}{\text{s}}$$

$$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$$

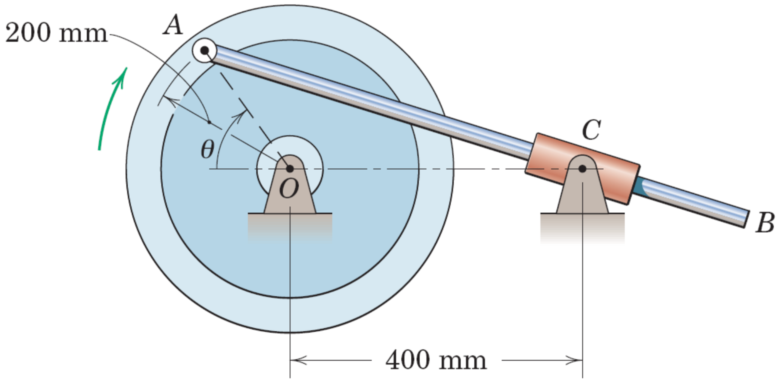
$$v_{P/O} = \overline{PO}\,\omega = 0.16(5.38) = 0.862 \ \frac{\text{m}}{\text{s}}$$

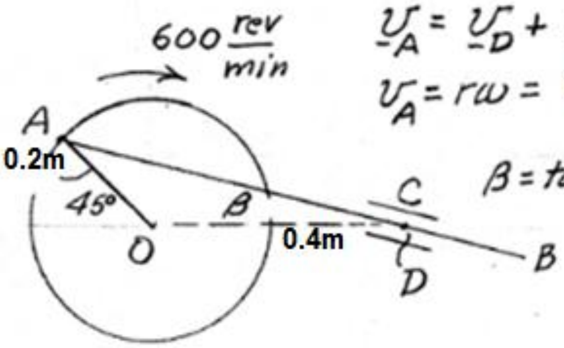



$$v_P = \sqrt{\overline{0.262}^2 + \overline{0.862}^2}$$
$$= \underline{0.900 \text{ m/s}}$$

PROBLEM 5/84

The flywheel turns clockwise with a constant speed of 600 rev/min, and the connecting rod $AB$ slides through the pivoted collar at $C$. For the position $\theta = 45°$, determine the angular velocity $\omega_{AB}$ of $AB$ by using the relative-velocity relations. (*Suggestion:* Choose a point $D$ on $AB$ coincident with $C$ as a reference point whose direction of velocity is known.)




$$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$$

$$v_A = r\omega = 0.2\,\frac{600(2\pi)}{60} = 12.56 \text{ m/s}$$

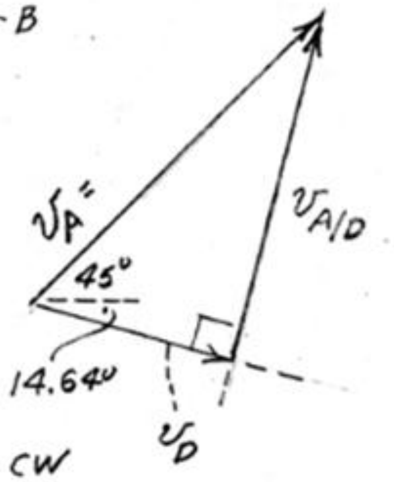
$$\beta = \tan^{-1}\frac{0.2\sin 45°}{0.4 + \cos 45°} = 14.64°$$

$$v_{A/D} = 12.56\sin(45° + 14.64°)$$
$$= 10.84 \text{ m/s}$$

$$\omega_{AB} = \omega_{AD} = \frac{v_{A/D}}{\overline{AD}}$$

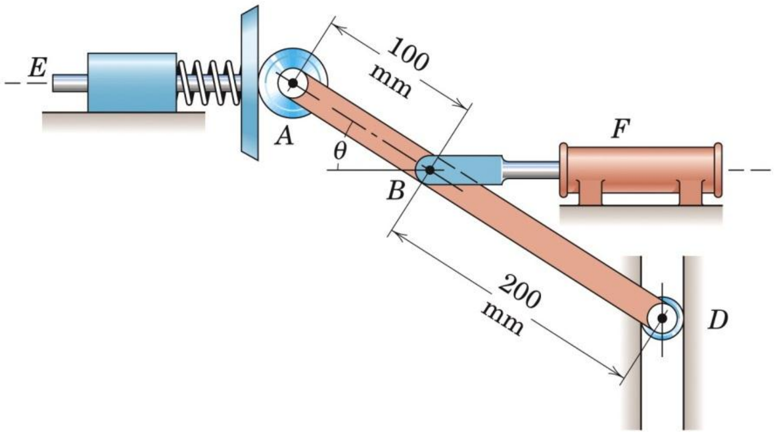
$$\overline{AD} = \frac{0.2\cos 45°}{\sin 14.64°} = 0.560 \text{ m}$$

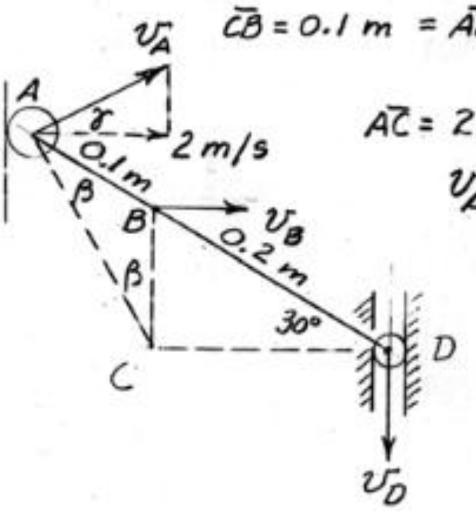
$$\omega_{AB} = \frac{10.84}{0.560} = \underline{19.38 \text{ rad/sec CW}}$$



PROBLEM 5/108

Horizontal oscillation of the spring-loaded plunger $E$ is controlled by varying the air pressure in the horizontal pneumatic cylinder $F$. If the plunger has a velocity of 2 m/s to the right when $\theta = 30°$, determine the downward velocity $v_D$ of roller $D$ in the vertical guide and find the angular velocity $\omega$ of $ABD$ for this position.




$\bar{CB} = 0.1\ m = \bar{AB}$ , $2\beta + (180-60) = 180$

$\beta = 30°$, $\gamma = 30°$

$\bar{AC} = 2(0.1)\cos 30° = 0.1732\ m$

$v_A = \frac{2}{\cos 30°} = 2.31\ m/s$

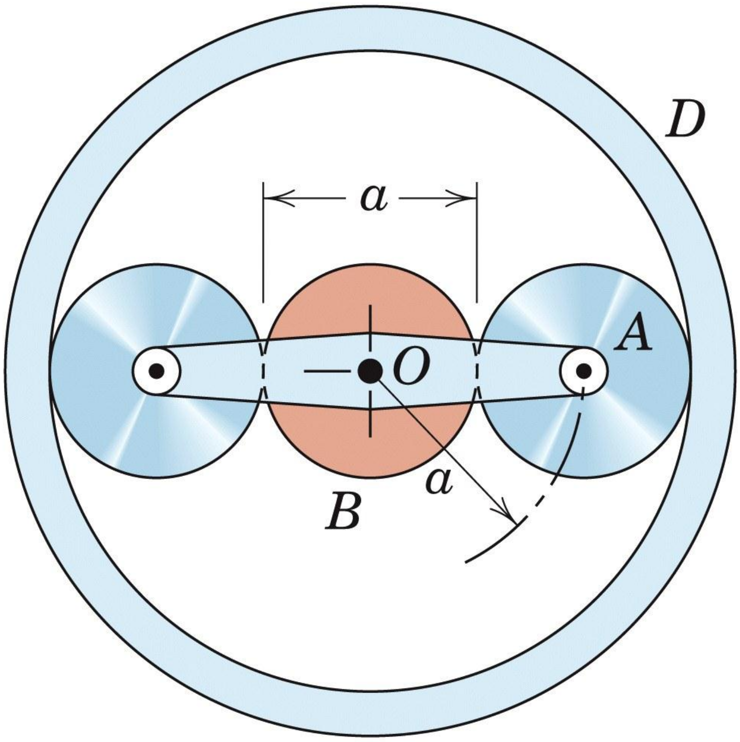
$\omega = \frac{v_A}{AC} = \frac{2.31}{0.1732} = \underline{13.33\ \frac{rad}{s}}$

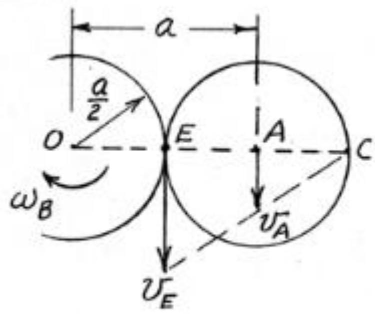
$v_D = \bar{CD}\omega = 0.2\cos 30°(13.33)$

$= \underline{2.31\ m/s}$

PROBLEM 5/120

The shaft at $O$ drives the arm $OA$ at a clockwise speed of 90 rev/min about the fixed bearing at $O$. Use the method of the instantaneous center of zero velocity to determine the rotational speed of gear $B$ (gear teeth not shown) if ($a$) ring gear $D$ is fixed and ($b$) ring gear $D$ rotates counterclockwise about $O$ with a speed of 80 rev/min.



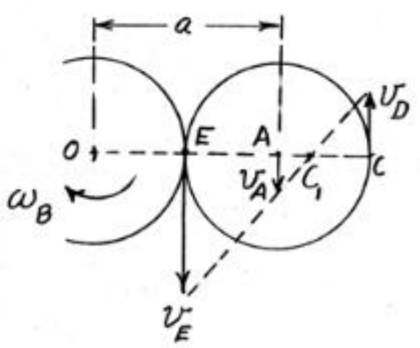




(a) $v_A = \omega_{OA}\, a$

$v_E = 2v_A = 2a\,\omega_{OA}$

$\omega_B = \frac{v_E}{a/2} = \frac{2a\,\omega_{OA}}{a/2} = 4(90)$

$= 360$ rev/min




(b) $v_D = \overline{OC}\,\omega_D = \frac{3a}{2}\, 80 = 120a$

$v_A = \overline{OA}\,\omega_{OA} = 90a$

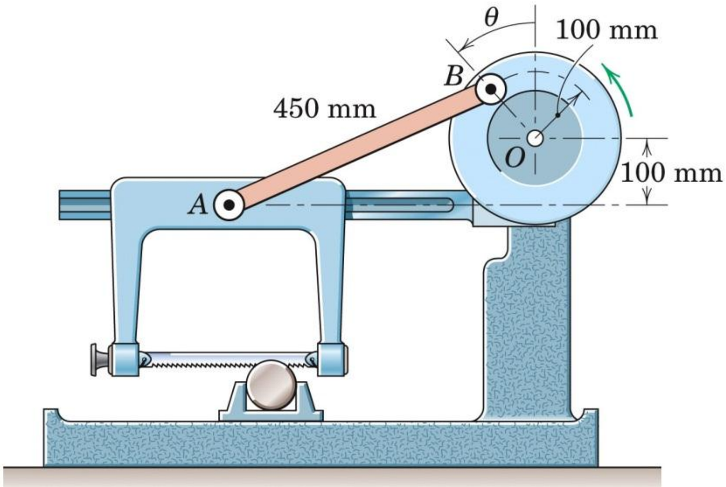
$\frac{v_A + v_D}{a} = \frac{v_E - v_A}{a}$ , $v_E = v_D + 2v_A$

$= 300a$

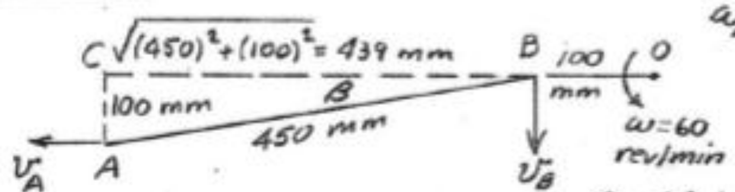
$\omega_{OE} = \omega_B = \frac{v_E}{a/2} = \frac{300a}{a/2} = 600\ \frac{rev}{min}$

PROBLEM 5/153

The elements of a power hacksaw are shown in the figure. The saw blade is mounted in a frame which slides along the horizontal guide. If the motor turns the flywheel at a constant counterclockwise speed of 60 rev/min, determine the acceleration of the blade for the position where $\theta = 90°$, and find the corresponding angular acceleration of the link $AB$.




5/153  C = instantaneous center of zero velocity for AB

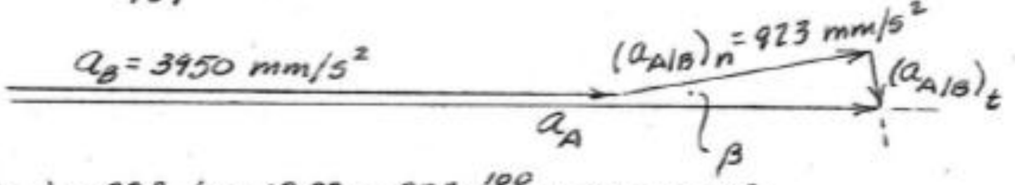



$$\omega_{AB} = \frac{v_B}{\overline{CB}} = \frac{\overline{OB}\,\omega}{\overline{CB}} = \frac{100}{439}\,\frac{60(2\pi)}{60} = 1.432\ \frac{\text{rad}}{\text{s}}$$

$$a_B = (a_B)_n = 100(2\pi)^2 = 3950\ \text{mm/s}^2$$

$$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t\,;\qquad (a_{A/B})_n = 450(1.432)^2 = 923\ \text{mm/s}^2$$

$$\beta = \tan^{-1}\frac{100}{439} = 12.82°$$




$$(a_{A/B})_t = 923 \tan 12.82 = 923\,\frac{100}{439} = 210\ \text{mm/s}^2$$

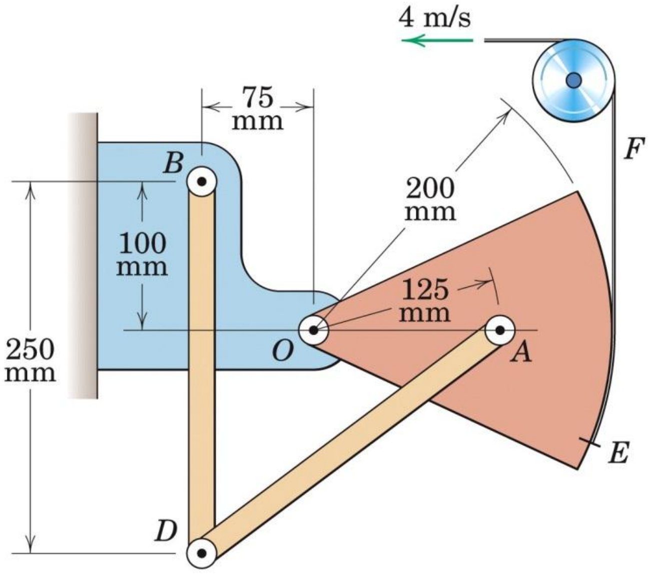
$$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{210}{450} = 0.467\ \text{rad/s}^2\ \text{CCW}$$

$$a_A = 3950 + 923\cos 12.82° + 210 \sin 12.82° = 4890\ \text{mm/s}^2$$

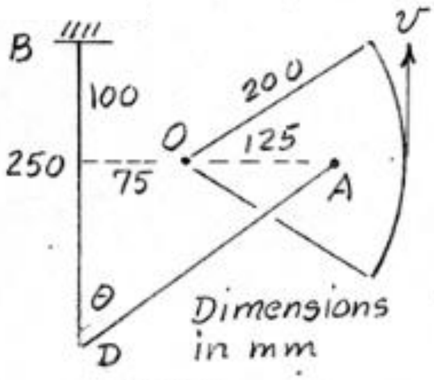
or $a_A = 4.89\ \text{m/s}^2$

PROBLEM 5/154

The mechanism of Prob. 5/115 is repeated here where the flexible band $F$ attached to the sector at $E$ is given a constant velocity of 4 m/s as shown. For the instant when $BD$ is perpendicular to $OA$, determine the angular acceleration of $BD$.




$v = 4$ m/s const.




$\theta = \tan^{-1}\frac{200}{150} = 53.1°$

$\overline{AD} = 250$ mm

From Prob. 5/118

$v_A = 2.5\ \frac{m}{s}$, $v_D = 1.875\ \frac{m}{s}$

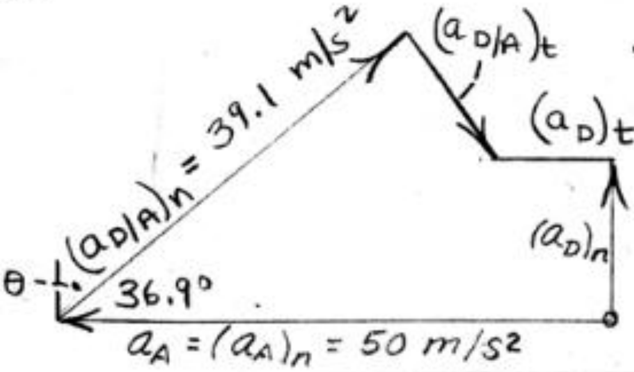
$\omega_{AD} = 12.5$ rad/s

$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A}$

$(a_D)_n = v_D^2/\overline{BD} = \frac{1.875^2}{0.250} = 14.06$ m/s$^2$

$a_A = (a_A)_n = v_A^2/\overline{OA} = \frac{2.5^2}{0.125} = 50$ m/s$^2$

$(a_{D/A})_n = \overline{AD}\,\omega_{AD}^2 = 0.250(12.5)^2 = 39.1$ m/s$^2$




Solution of polygon gives $(a_{D/A})_t = 11.72$ m/s$^2$

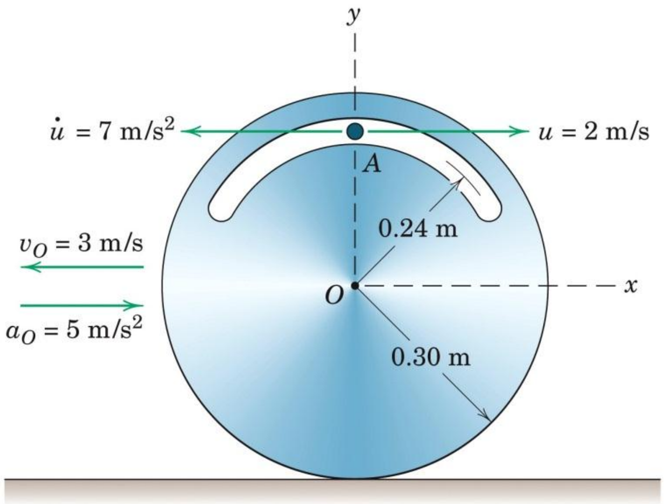
$(a_D)_t = 11.72$ m/s$^2$

$\alpha_{BD} = (a_D)_t/\overline{BD}$

$= \frac{11.72}{0.25} = 46.9\ \frac{rad}{s^2}$ CW

PROBLEM 5/162

The disk rolls without slipping on the horizontal surface, and at the instant represented, the center $O$ has the velocity and acceleration shown in the figure. For this instant, the particle $A$ has the indicated speed $u$ and time-rate-of-change of speed $\dot{u}$, both relative to the disk. Determine the absolute velocity and acceleration of particle $A$.




For the coordinates $\circlearrowleft^{+}$ $y$, $x$, the no-slip constraints are $v_o = -r\omega$ & $a_o = -r\alpha$. So

$$\omega = -\frac{v_o}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_o}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame $Oxy$ as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_o + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_o + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients:

$\underline{v}_o = -3\underline{i}$ m/s

$\underline{a}_o = 5\underline{i}$ m/s²

$\underline{\omega} = 10\underline{k}$ rad/s

$\underline{\alpha} = -16.67\underline{k}$ rad/s²

$\underline{r} = 0.24\underline{j}$ m

$\underline{v}_{rel} = 2\underline{i}$ m/s

$\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j}$

$= -7\underline{i} - 16.67\underline{j}$ m/s²

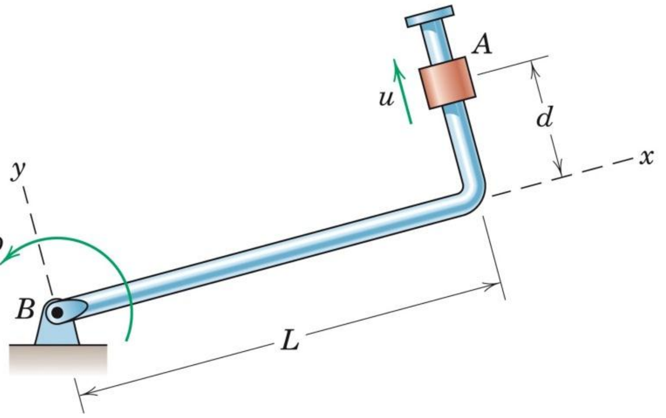
Substitute into (5/12) & (5/14) & simplify:

$\underline{v}_A = -3.4\underline{i}$ m/s

$\underline{a}_A = 2\underline{i} - 0.667\underline{j}$ m/s²

PROBLEM 5/165

The small cylinder $A$ is sliding on the bent bar with speed $u$ relative to the bar as shown. Simultaneously, the bar is rotating with angular velocity $\omega$ about the fixed pivot $B$. Take the $x$-$y$ axes to be fixed to the bar and determine the Coriolis acceleration of the slider for the instant represented. Interpret your result.

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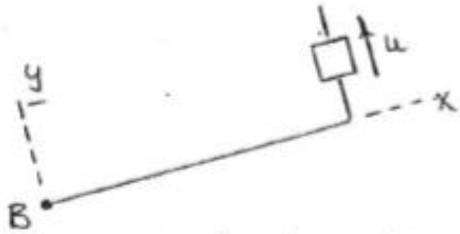
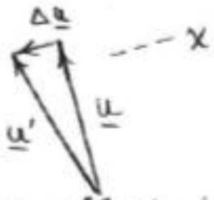
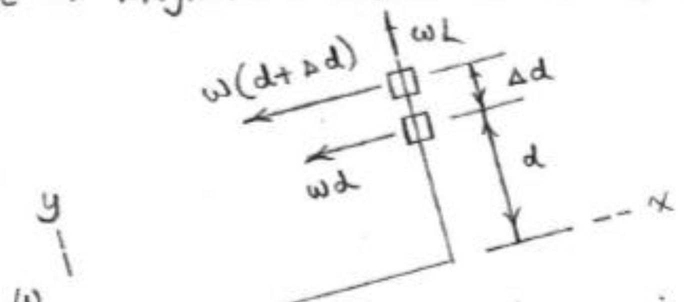
$$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}_{rel}$$
$$= 2\omega\underline{k} \times u\underline{j} = \underline{-2\omega u\underline{i}}$$




Change-of-direction effect is in $-x$ direction:




Change-of-magnitude effect is in $-x$ direction:




(ωL in y-direction unchanged by Δd.)